

# Active Guidance and Dynamic Flight Mechanics for Model Rockets

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*Annotated and abridged by Duncan McDonald (TRA 7311) 2003  
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## Forward

Although written to motivate the discussion of rocket guidance, as an introduction to rocket aerodynamics it is the best article I have ever read. Unfortunately the notation that was used in the original article was BASIC computer code, which made it both difficult to read and impossible to compare to other aerodynamic articles, books, or computer programs like RockSim or FinSim. There is a lot of excellent information here! I rewrote it exactly as originally written but changed the notation to NACA (the precursor to NASA) standard notation. I also added a few notes and amplifications in blue.

David Ketchledge: I tried many times through many different avenues to contact you and get your permission before I posted this, but you seem to have disappeared. If you object to this posting contact me and I will remove it immediately.

## Introduction

Model rocketry and High Powered Rocketry have to date relied upon the static ability of a model or vehicle in order to maintain a vertical flight. In our recent history a few cases of active guidance have been performed between 1988 and 1992. The paper written and published on sun sensor guidance dealt in great depth with the technical end of the associated sensors and electronics. (*The Tripolitan, America's High Power Rocketry Magazine*, Volume 7 Number 2, May/June 1992 and Number 3, July/August 1992) [now available from NAR Technical Reports TR-204. See reference 1]. However, it did not discuss Flight Mechanics in any shape or form which is a prerequisite to designing proper guided vehicles of moderate or high performance. It is the hope of this author that , through his work, a practical design tool will allow the inclusion of vertically guided vehicles into the art of model and high performance rocketry.

We may ask why does model and high performance rocketry require any form of guidance? In the vast majority of flights active guidance is not required. However, if one is going to construct a vehicle that can attain altitudes of over 50,000 feet and as high as 100,00 feet active guidance is a requirement. Our present method is to have an extremely high thrust and short burn time. This, in turn, causes the given vehicle to go transonic and then supersonic very near the ground where air density is high. In some cases we see fin flutter and fins failing followed by the craft breaking up in mid air. Simply stated, it would seem that we have found the far end of the flight envelope. In addition, about 60%

of motor thrust is lost to drag. This rapid burn method is not an effective use of propellant. The most effective thrust time curve is one with an initial peak, a long sustaining burn of 10 to 20 seconds and then ending with a final peak. From an engineering point of view this we would call a progressive-neutral-progressive burn pattern. Motor manufacturers such as NCR, FSI, ISP have produced motors of this type and are continuing to investigate them. Why are they so interested in long burn time motors? Because these motors offer the best altitude performance. In fact, these motors may allow high powered rocketry the ability to reach nearly into space in the coming decade. If we fly a motor that will operate for 20 seconds we face a flight mechanics problem. Simply, we will find that a statically stable vehicle will turn into the wind. Why are there no four stage model rockets? Because weather cocking effects have never been able to be removed. Even a three stage rocket experiences a large amount of the weather cocking effect. Clearly, we must find a better set of solutions.

The advent of high power rocketry has allowed a growth in payload weight that previous rockets could not lift. Since it is not uncommon today to see rockets flying in the range of .75 to several hundred pounds, it is not a case of weight but engineering application that prevents active guidance in model rocketry. Methods using an optical pendulum system or R/C gyro (found in R/C helicopters) *can* provide guidance systems when coupled to guidance fins.

First and foremost we must not produce a vehicle that is a targeting guided missile. The regulatory area is very clear in this. The NAR safety code does not allow for a flight past 30° of vertical and the systems found in this article will maintain within 5° of vertical. In addition, the systems discussed are only for the elimination of gravity turning and weather cocking and not an inertial guidance system that allows for the ability to seek targets, fixed or moving, on the ground or in the air. At no time does the author condone any form of disregard to NAR safety codes or those of the Tripoli Rocketry Association. The technology of this article is a design tool for vehicle designers in order to allow flights in a vertical trajectory only.

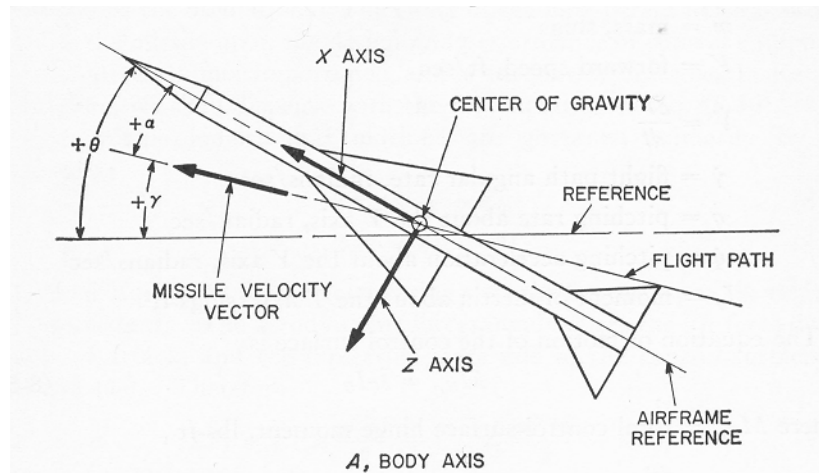
Before we develop a vertical trajectory system (VTS) we must reopen the classic works of Barrowman calculations on stability [[available online at Apogee Rockets. See reference 2](#)]. These solutions, available since the late 1960's, treated vehicles in a static manner, but a new set of tools can now be applied using computer modeling. However, we need a dynamic model of a rocket in flight which goes beyond the Barrowman methods. An excellent treatment can be found in the college textbook, *Aerodynamics and Flight Mechanics* by McCormic, ©1979 Wiley Publishing. By applying the Barrowman Normal Force Coefficients and the McCormick methods, we can model a dynamic flying environment. In addition, the method will allow for the treatment of any vehicle. In the end, an individual with a good understanding of algebra will be able to construct a computer model for a given vehicle and determine fin locations and sizes for VTS system. Also, this method will allow for flight speeds in the subsonic to supersonic regions.

First we will go into the fundamentals of *Flight Mechanics* as it pertains to model and high power rocketry. Then we will advance to a discussion of controller theory for taking

sensor data and producing a set of signals to servos. From here the next logical step will be a complete computer model of a vehicle comprising its flight dynamics and controller system (VTS) [As far as I know, the original code is lost to history. There is similar code available for download that was written by Steve Ainsworth, another high powered rocketry pioneer. See reference 3]. Lastly, we will study the results and make conclusions on the design aspects and requirements for a good design.

## Flight Mechanics for Model Rockets in a Dynamic Model

### NOTE 1



(Figure taken from “Principles of Guided Missile Design Vol. 7 Systems Preliminary Design” by J.J. Jerger. Van Nostrand 1960, page 405)

Where:

$\alpha$  is the angle of attack

$\gamma$  is the flight path angle

$\theta$  is the pitch angle

Reference is the horizon

Before we do anything else lets talk about angle of attack, represented by the Greek letter alpha ( $\alpha$ ), one of the most important and subtle concepts in aerodynamics. Well, what is angle of attack? If a rocket is not being acted on by any external force and there is no thrust misalignment, the velocity vector (trigonometric sum of all velocity components, i.e. forward, sideways, up, down) will line up with the airframes longitudinal axis. If there is an external force like wind, there will be an angle between the velocity vector and the airframe because the rocket is being pushed slightly to the side. That angle is the angle-of-attack,  $\alpha$ . A rocket that is passively controlled by fins will always try and fly at

zero angle of attack. Anytime a force like side wind or wind shear causes any non-zero angle of attack to appear, the fins will instantly try and rotate the rocket back into the airstream and reduce the angle of attack to zero. So if it is windy, wind makes up part of the velocity vector and the rocket will turn into the wind. In a passively guided (i.e. fixed fins only) rocket, it takes an external force to create alpha. The angle of attack is generally highest right when a rocket leaves the launch rail and again after motor burnout when the rocket has substantially slowed down. Because velocity is increasing in the early stages of the flight and dynamic pressure is highest then, an induced angle of attack can cause tremendous forces to be put on the fins. That is the one of the main reasons rockets tend to shred at very low altitudes.

Let's look at the two dimensional example shown above. For a coordinate system centered in the rocket (we'll actually use three coordinate systems in the course of this article: body-fixed, a coordinate system centered in the rocket; space-fixed, a coordinate system centered in space or on the ground; and wind-axes, a coordinate system fixed in the airstream), the angle between the rocket and the horizon is called the pitch angle represented by the Greek letter theta ( $\theta$ ). The angle between the rocket and the airstream (that is, the airstream as viewed by the rocket) is alpha and the angle between the airstream and the horizon is called the flight path angle represented by the Greek letter gamma ( $\tilde{\alpha}$ ). Why is the flight path angle  $\tilde{\alpha}$  and not  $\theta$ ? Because the rocket is actually moving along the velocity vector, but at a slight angle. When a rocket takes on a non-zero angle of attack it is actually skidding through the air! But because the angle of attack is generally fairly low the skid is actually a slight one. Unlike airplanes, rockets skid to turn instead of banking like an airplane to turn.

Look at the sections on Coordinate System and Oscillation and Rotation Damping to see how angle of attack looks in three dimensions and for more on frames of reference.

Before continuing, we need to review the Barrowman method and apply its equations for a dynamic flight model. This model must consider that a vehicles velocity, angle of attack, mass and normal force coefficients are in a constant state of change and not static. Also, this model must account for subsonic compressible flow, transonic and supersonic air flows. If one closely reads the original Barrowman works, it becomes clear that the availability of a computer, at the time of his study, would have allowed his equations to be applied in a dynamic model. Through his classic work will come the majority of the model to be presented.

Barrowman's method is based on the concept of the normal force coefficient,  $CN_{\tilde{\alpha}}$ , a dimensionless number dependent upon the shape of the rocket which permits the calculation for the force action in the direction perpendicular to the rocket's longitudinal axis whenever it is displaced from the direction of the relative air stream. The equation that accomplishes this is as follows:

$$(1-1) \quad F_N = \frac{1}{2} \rho V^2 a C_{Na} S_R$$

Where:

$F_N$  is the normal force

$\rho$  is the mass density of the air

$V$  is the Airspeed of the rocket

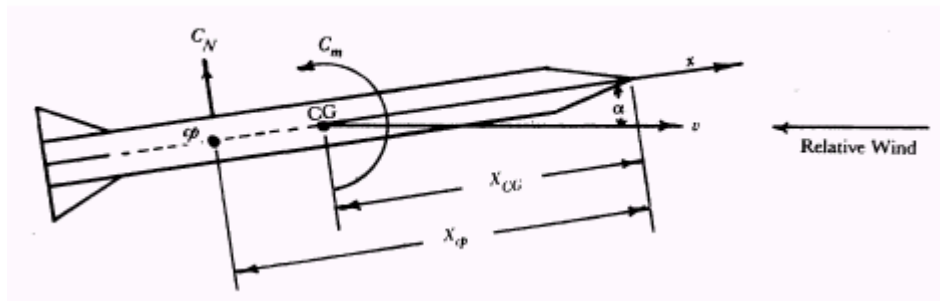
$S_R$  is the reference area or cross-sectional area

$a$  is the angle of attack in radians

Now the normal force coefficient ( $C_{Na}$ ) of the rocket considered as a whole is the sum of the normal force coefficients of the individual components of which it is composed. Each part is thus considered to act as a point on the component called the center of pressure (CP). The method uses this technique of sectionalized analysis, together with the theory of moments to derive the total normal force coefficient and center of pressure of the complete rocket.

#### NOTE 2

The basic idea is that a force develops, due to the angle of attack, and acts on the center of pressure of the rocket and causes it to turn about the center of gravity. The force with which it turns is a torque or moment.



(from Design of Aerodynamically Stabilized Free Rockets Mil-Hdbk-762(MI) Department of Defense 1990.)

$$m = \bar{X} F_N$$

That is, a moment  $m$  is created around the center of gravity by the application of the normal force  $F_N$  at the center of pressure that gets multiplied by a moment arm  $\bar{X}$  that is the distance between the center of pressure and the center of gravity.

Forces due to the angle of attack can be described as a product of the angle of attack, the aerodynamic pressure  $q$ , and the particular *stability derivative* for the force being considered.

$$q\mathbf{a}C_{ma} = \bar{X}(q\mathbf{a}C_{Na})$$

Where:

$$q = \frac{1}{2} \rho V^2 \text{ is the aerodynamic pressure}$$

$$C_{ma} = \bar{X}C_{Na}$$

$$F_N = q\mathbf{a}C_{Na} \text{ and}$$

$$m = q\mathbf{a}C_{ma} .$$

Coefficients that start with a capital “C”, like  $C_{ma}$  and  $C_{Na}$  are called *stability derivatives*. So that different fin and airframe geometries can be compared to each other independent of their actual size, the forces and moments are normalized to a dimensionless number, the stability derivative. Unfortunately stability derivatives tend to be nonlinear, depending on airspeed, Reynolds number, and angle of attack.

The moment stability derivative for the pitch axis can be described by the summation of the products of the normal force stability derivatives for each piece of the rocket and the distance from that pieces center of pressure to the overall rockets center of gravity.

$$C_{ma}^{total} = \bar{X}_{nose} C_{Na(nose)} + \bar{X}_{shoulder} C_{Na(shoulder)} + \bar{X}_{boattail} C_{Na(boattail)} + \bar{X}_{fins} C_{Na(fins)}$$

In the dynamic model, we will use the component part center of pressures and the magnitude of the normal force in order to generate moments and rotations of the complete rocket. Unlike the Barrowman method, we will correct the normal force for compressibility of the air above .42 Mach and for the transonic and supersonic regions up to about Mach 3. Like the Barrowman method we will not allow the angle of attack to exceed .2 radians or 11.46°. We will assume that the fins are thin, flat airfoils and do not deflect or bend.

[Table inserted by editor]

Axis		Force Along Axis			Moments About Axis			Velocities	
<i>Designation</i>	<i>Symbol</i>	<i>Designation</i>	<i>Symbol</i>	<i>Positive Direction</i>	<i>Designation</i>	<i>Symbol</i>	<i>Positive Direction</i>	<i>Linear</i>	<i>Angular</i>
Longitudinal	X	Longitudinal	F <sub>X</sub> or X	Y→Z	Roll	<i>l</i>	Y→Z	<i>u</i>	<i>p</i>
Lateral	Y	Lateral	F <sub>Y</sub> or Y	Z→X	Pitch	<i>m</i>	Z→X	<i>v</i>	<i>q</i>
Normal	Z	Normal	F <sub>Z</sub> or Z or F <sub>N</sub>	X→Y	Yaw	<i>n</i>	X→Y	<i>w</i>	<i>r</i>
Axis		Angles (Rocket referred to ground)* a.k.a. Euler Angles		Angles (Rocket referred to Airstream)**		Moments of Inertia			
<i>Designation</i>	<i>Symbol</i>	<i>Designation</i>	<i>Symbol</i>	<i>Designation</i>	<i>Symbol</i>	<i>Designation</i>	<i>Symbol</i>		
Longitudinal	X	Roll	<i>j</i>	Roll	<i>f</i>	Longitudinal	<i>I<sub>XX</sub> or I<sub>L</sub></i>		
Lateral	Y	Pitch	<i>q</i>	Pitch	<i>a</i>	Lateral	<i>I<sub>YY</sub></i>		
Normal	Z	Yaw	<i>y</i>	Yaw	<i>b</i>	Normal	<i>I<sub>ZZ</sub></i>		

**Table 1 NACA Standard Notation**

\* This coordinate system is also called a “space-fixed” system. The angles that relate a space-fixed system to a body-fixed system are known as Euler angles.

\*\* This coordinate system is also called a “body-fixed” system.

## Nose Cone Normal Force Coefficient and CP

We start off by looking at the standard Barrowman nose cone center of pressure locations illustrated in Figure 1, below

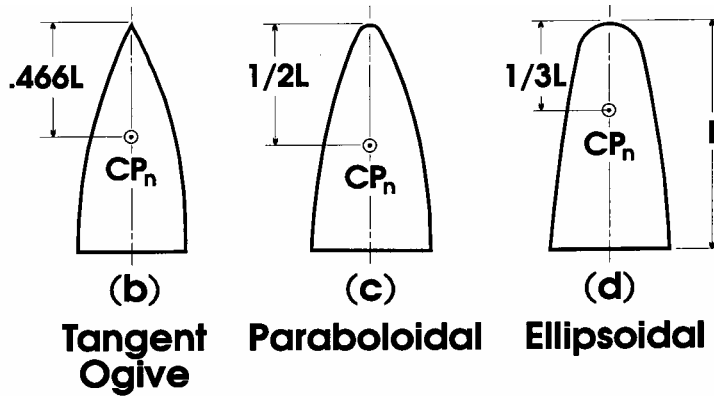


Figure 1

Under these conditions it can be shown that the normal force coefficient of the nose cone is independent of its shape.

In the Barrowman method this is as follows:

$$(1-2) \quad C_{Na(nose)} = 2.000$$

Where “nose” denotes the nose cone

The location of the nose cone center of pressure is found by specialization of the more general relationship determined in Barrowman’s paper. That is to determine the location of the CP of an axially symmetrical section of the rocket by first computing the volume enclosed by its surface. Then divide this volume by the area of the base (cross sectional area of the component at its greatest diameter). The result from this division will be in units of length. Starting from the position of the base, traveling a distance equal to this length in the direction of the distance equal to this length in the direction of the component’s taper, the point you will locate in this manner is the component’s center of pressure (CP).

### NOTE 3

In general, the center of pressure of a nosecone can be found by

$$\bar{X}_{nose} = l \left( 1 - \frac{V_{nose}}{\rho l r^2} \right)$$

Where:

$l$  is the length of the nosecone

$r$  is the radius of the nosecone

$\bar{X}_{cp(nose)}$  is the nosecone center of pressure

This allows us to use any part that can have a computed volume such as nose cones, transitions, boat tails and fins. If the volume cannot be computed by a function, the part may be immersed in water (in a graduated beaker) to determine the volume also. Thus, if one has a custom part, you can still determine its center of pressure. Most nose cones, however, have equations that give the center of pressure and are stated below:

$$(1-3) \quad \bar{X}_{cp(nose)} = \frac{2}{3}l \quad \text{Conical Nose}$$

$$(1-4) \quad \bar{X}_{cp(nose)} = 0.466l \quad \text{Tangent Ogive Nose}$$

$$(1-5) \quad \bar{X}_{cp(nose)} = \frac{1}{2}l \quad \text{Paraboloidal Nose}$$

$$(1-6) \quad \bar{X}_{cp(nose)} = \frac{1}{3}l \quad \text{Ellipsoidal Nose}$$

### Transition Normal Force Coefficient and CP

To determine the normal force coefficient ( $C_{Na}$ ) of a transition or shoulder is as follows:

$$(1-7) \quad C_{Na(shoulder)} = 2 \left( \left( \frac{R_1}{R_2} \right)^2 - \left( \frac{R_1}{R_r} \right)^2 \right)$$

Where:

$R_2$  is the base radius

$R_1$  is the upper shoulder radius

$R_r$  is the reference radius

The location for the center of pressure of the conic shoulder is also given by the following:

$$(1-8) \quad \bar{X}_{(shoulder)} = X_{shoulder} + l \left( \frac{2}{3} - \frac{1}{3} \left( \frac{R_1}{R_2} \right) \left( \frac{R_1}{R_2} + 1 \right) \right)$$

Where:

$X_{shoulder}$  is the distance from the nose cone to the start of the shoulder

$l$  is the length of the shoulder

$R_1$  is the upper shoulder radius

$R_2$  is the lower shoulder radius

If we use a conical boat tail, the normal force coefficient equation is identical to that used for the shoulder. However, in this case  $R_2$  is lower than  $R_1$ , meaning that the value of  $C_{Na(shoulder)}$  is negative in value. In a practical sense, a boat tail generates a suction force as the rocket moves through an air stream. The center of pressure of the boat tail is given by:

$$(1-9) \quad \bar{X}_{(boattail)} = X_{boattail} + \frac{l}{3} \left( 1 + \left( \frac{R_2}{R_1} \right) \left( \frac{R_2}{R_1} + 1 \right) \right)$$



#### NOTE 4

(1-10) considers the force acting on one half of a pair of coplanar fins in one force direction, the pitch direction. There are two planes: the pitch plane and the yaw plane. So for one plane  $n$  is 2 and for two planes (the total force)  $n$  is 4 for a four fin rocket. For a three fin rocket see Barrowman (see reference 2).

Where:

$n$  is the number of fins

$l$  is the mid-span length

$C_r$  is fin root cord

$C_t$  is the fin's tip cord

$S$  is the fin span

$d$  is the body tube diameter

We need to account also for the interference of the rocket's body [where] the fin attaches. This can be found using the following function:

$$(1-11) \quad K_{tb} = 1 + \frac{1}{\left( \frac{(S + R_t)}{R_t} \right)}$$

We then get the corrected normal force coefficient  $C_{Na_{tb}(fin)}$  as:

$$(1-12) \quad C_{Na_{tb}(fin)} = (K_{tb})(C_{Na(fin)})$$

The center of pressure of the fins then follows as:

$$(1-13) \quad \bar{X}_{(fin)} = X_{finroot} + \frac{X_t}{3} \left( \frac{C_r + 2C_t}{C_r + C_t} \right) + \frac{1}{6} \left( (C_r + C_t) - \left( \frac{C_r C_t}{C_r + C_t} \right) \right)$$

Where:

$X_t$  is the distance of the fin root leading edge to the fin tip leading edge laterally

$X_{finroot}$  is the distance from the nose cone tip to the start of the fin

$C_r$  is the fin root cord

$C_t$  is the fin tip cord

The radial position of the fins center of pressure is given by:

$$(1-13a) \quad \bar{Y}_{(fin)} = r_t + \frac{S}{3} \left( \frac{C_r + 2C_t}{C_r + C_t} \right)$$

Where:

$r_t$  is the body tube radius

This now completes the determination of the normal force coefficients and related center of pressures. In our dynamic model we are not concerned where the overall vehicle center of pressure is located, but only the components parts of the vehicle. Therefore, the values of  $X_{finroot}$  and  $X_{boattail}$  are taken to be zero in the dynamic model. The normal force coefficients are each multiplied in equation (1-1) to derive the normal forces and related torques applied to the craft. In this manner, using the equations as a basis, we can determine the rotations, translation forces and behavior of a vehicle. As a design of a VTS style vehicle that is not only statically stable but removes the effects of weather cocking and gravity turns prior to vehicle construction. However, we must again correct the value of the normal force coefficients when the vehicle flies in the subsonic compressible, transonic and supersonic regions of flight.

### **Subsonic Compressible Region**

Physically, the subsonic compressible region occurs when the air ahead of the vehicle during its flight starts to compress due to the craft's motion. From the reference text, this point is approximately Mach .42 and above. It now becomes necessary to correct the

functions of pitching moment, lift and drag for this compression of the air even though the craft is flying in the subsonic region of speeds. Though the free stream Mach number is below 1, the airflow over the fin may exceed the local speed of sound and form a shock wave. Tapering a fin will reduce the chance of the fin exceeding sonic airspeed over its surface by the fact that the airflow normal to the fin is less than the free stream Mach number.

Since small pressure disturbances travel at the speed of sound, the time the fluid particle ahead of the moving body is influenced by the pressure field around the body is proportional to the difference between the acoustic velocity and the velocity of the vehicle's aerodynamic surfaces. As the vehicle approaches a free stream mach number of unity, the fluid (air) ahead of the craft is displaced less and less. As long as the free stream Mach is below unity, a factor termed the "Prandtl-Glauert compressibility correction factor"  $\mathbf{b}$  can be used to correct for the lift, drag and pitching coefficients of the fin and rudder of the vehicle. It should be noted that the centers are not affected by this compression of the air. Mathematically, the expression for the compressibility factor  $\hat{a}$  is as follows.

$$(1-14) \quad \mathbf{b} = \sqrt{(1 - M_0^2)}$$

Where:

$M_0$  is the free stream Mach number

Dividing the previous equations for the normal force coefficient factor equation yields the proper effect on the nose, shoulder, boat tail sections and fins. These components will undergo approaches the transonic region. The equations below illustrate this:

$$(1-15a) \quad C_{Na(b(fin))} = \frac{C_{Na(b(fin))}}{\sqrt{(1 - M_0^2)}}$$

$$(1-15b) \quad C_{Na(nose)} = \frac{C_{Na(nose)}}{\sqrt{(1 - M_0^2)}}$$

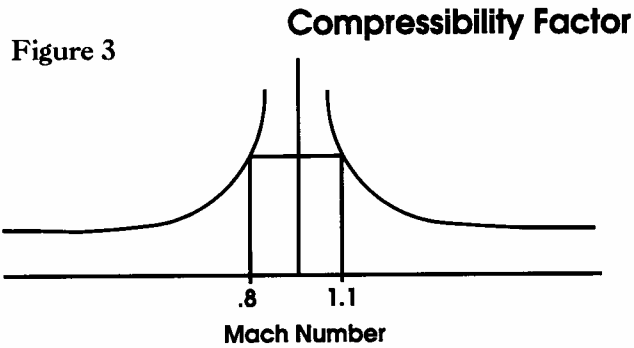
$$(1-15c) \quad C_{Na(shoulder)} = \frac{C_{Na(shoulder)}}{\sqrt{(1-M_0^2)}}$$

In regard to the computer simulation model for this region, the craft's resultant velocity must fall between Mach .42 and Mach .8 in order to undergo this correction factor. Above .8 Mach in order to undergo this correction region where shock waves will develop on the vehicles nose, shoulder, boat tail and fins. Here the compressibility factor becomes asymptotic and no longer fits the actual fin or rudder condition thus another method of approach is necessary to solve for the parameters of normal force coefficients.

### **Transonic Flow Model**

Within the transonic flow region, the fin surfaces will have local areas of flow that become sonic and thus develop shock waves. During the years prior to aircraft being able to exceed the sound barrier, pilots would find that their vehicles would buffet wildly or lose control as they approached the sound barrier. Many flew to their deaths because, when in a dive, their planes would reach the transonic region and the flight characteristic of their planes would become uncontrollable. The problem, of course, was resolved once the cause of the problem was understood. The shock wave, developed by the fin, rudder or stabilizer, would make the portion of the surface aft of the wave virtually nonexistent in terms of equations of flight mechanics. In engineering terms, the developed shock wave produces a turbulent wake aft of the shock wave which, in turn, causes the control surface to become ineffective. At slightly higher mach numbers still within the transonic region, the shock wave will move off the aerodynamic surface and another shock wave will form that is parallel to the chord (and is connected to the initial shock wave). This situation aggravates the loss of the control surface's ability to cause changes in the vehicles angle along the primary axis.

Analysis of aerodynamic behavior in the transonic region of velocities is difficult. The reference text gives only other [texts] that treats the subject area specifically. In regard to a vehicle's fin profiles are that of the NACA 64A009 airfoil which the reference text provides several curves for its transonic behavior. Since there are no applicable functions for the transonic region, it is necessary to approximate the behavior of the fin in the transonic region. The graph (Figure 3) illustrates this process.



**Figure 3**

The line of intersection that was picked starts at .8 mach and extends to Mach 1.1. Once the free stream Mach number reaches a value of 1.1, the vehicle is considered to be under the equations for supersonic flow that will be discussed later in this article. In mathematical terms, the value of the compressibility factor is fixed to a value of

$$0.6 = \sqrt{1 - (0.8)^2}$$

and is held to this value until the supersonic region is achieved at Mach 1.1.

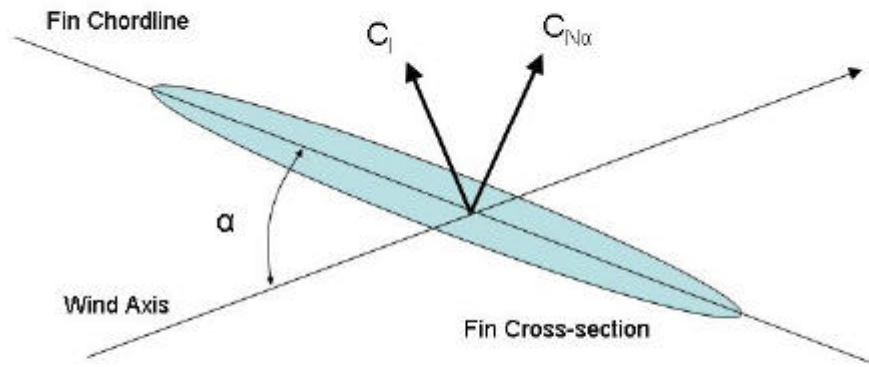
### **Supersonic Flow Model**

It is not the intent of this article to go into the lengthy differential equation theory of the Ackeret linearized flow model of supersonic fins, but to present the resultant equations that are developed from the reference text and discuss the limitations approach yields results that are somewhat optimistic with regard to the lift and drag coefficients as well as its prediction of the center of pressure. Yet, mathematically, the approach is good at following the trend as vehicle velocity increases. Approximate error for the model in regard to the lift coefficient is about 10%, 20% for the coefficient of drag and 19.5% for the center of pressure location. The model uses the following functions that are applied in the computer model for Mach numbers from 1.1 to Mach 3.

The lift coefficient equation in the Ackeret linearized model follows the actual fin lift coefficient closely as long as the angle of attack is kept small in value with a range from zero to five degrees. Above five degrees angle of attack, the model diverges exponentially from the actual lift curve. The function, below, generates the lift coefficient:

$$(1-16) \quad C_l = \frac{4a}{\sqrt{(M_0^2 - 1)}}$$

NOTE 5



Equation 1-16 is true for a coordinate system fixed in the airstream (this coordinate system is sometimes called the “wind-axes”), but we have been using a body-fixed coordinate system. Lift is defined as that component of force which is normal to the direction of the airstream and as you can see in the illustration above,  $C_l$  is normal to the airstream (wind axis).  $C_{Na}$  on the other hand is normal to the fin chordline so  $C_l$  and  $C_{Na}$  are separated by the angle of attack. Ignoring drag, if we convert this to the body-fixed coordinate system we have been using we get:

$$C_{Na} = \frac{4a}{\sqrt{(M_0^2 - 1)}} \quad \text{and}$$

$$C_l = C_{Na} \cos \alpha$$

In brief, this completes the equations for the interaction of the fins with the air stream flowing over the rocket. Normally the angle of attack  $\alpha$  is zero or nearly zero. However, when the angle of attack does take on a non-zero value, the fins will generate lift to rotate the overall vehicle. For this reason, the development of the equations (1-1) through (1-31) were required so that changes in the direction of the vector angle of air velocity (due to cross winds that lead to weather cocking) can be accounted for in a computerized model.

### Flight Mechanics Analyzer

We are now ready to determine the torques and rotation of a vehicle in terms of a mathematics framework. When a fin generates lift forces there must be some positive angle of attack. We can then say, when the vector of air velocity over the fin takes on a non-zero value, the fin will generate lift (or a normal force) and thereby torque. This torque applied to the center of gravity will turn the vehicle. This is why our vehicles weathercock and it is also what we will use in a VTS system in order to counteract the effect of weather cocking and gravity turning. By rotating a set of fins, positioned near the nose cone, forces and torques to correct the weather cocking effect and gravity turning can occur. The forces generated by the fins must take into account fin aspect ratio, vehicle velocity and the behavior of the air in terms of it being subsonic and possibly supersonic.

Using the equations that we have developed for the fin's lift (corrected for aspect ratio and Mach number) we need only determine the angular difference of the vehicle when compared to the vector direction of the airflow over the vehicle. This angle will generate fin lift and, thus, the forces to rotate the vehicle. To determine the attitude of the vehicle, we use the vehicle's longitudinal moment of inertia  $I_L$ . This can be computed using the following:

$$(1-17) \quad I_L = \sum \left( \frac{W_c X_{cg}^2}{g} \right)$$

Where:

$W_c$  is the weight of the components in pounds

$X_{cg}$  is the distance from the center of gravity of the overall vehicle

$g$  is 32.174, the value of gravity at sea level in fps squared

This equation will generate values that are somewhat lower than the actual amount of the longitudinal moment of inertia since it does not consider the  $I_L$  values of the parts themselves. A more precise method is by using a tension balance on the model rocket. A tension balance is simply a long wire attached to the center of gravity and a known force applied to the vehicle. An excellent discussion on how to calibrate and use such a device can be found in Advanced Topics of Model Rocketry (MIT Press) [unfortunately long out of print. I saw a copy for sale once for \$250. If someone has a copy I would love to read it!]. Our approximation will work for us to get the majority of the moment of inertia and will omit the extensive equations for each geometry of the parts.

We have now concluded our discussion of vehicle normal force coefficients on the nose and fins of a model rocket. In summary, we now can commute the normal force coefficients and, if the angle of attack and vehicle velocity is known, we will then find the magnitude of force generated by each component. The equations given below express this:

$$(1-18) \quad F_{N(nose)} = \frac{1}{2} \mathbf{r} V^2 C_{Na(nose)} \mathbf{a}$$

$$(1-19) \quad F_{N(fins)} = \frac{1}{2} \mathbf{r} V^2 C_{Na(fins)} \mathbf{a}$$

$$(1-20) \quad F_{N(canards)} = \frac{1}{2} \mathbf{r} V^2 C_{Na(canards)} (\mathbf{a} + \mathbf{d})$$

Where:

$C_{Na(nose)}$  is the nose normal force coefficient (2.000)

$C_{Na(fins)}$  is the normal force coefficient for the fins at the base of the rocket

$C_{Na(canards)}$  is the normal force coefficient for the guidance fins of the rocket

$\mathbf{r}$  is the air density

$V$  is the vehicle velocity in Feet/second

$\mathbf{a}$  is the angle of attack in radians

$\mathbf{d}$  is the servo driven angle of a guidance fin in radians

$F_{N(nose)}$  is the nose cone force

$F_{N(fins)}$  is the force generated by the base fins

$F_{N(canards)}$  is the force generated by the guidance fins

We still need to be able to convert these forces into torques and then rotations about the center of gravity for the vehicle under study. To mechanical engineers, this is a simple case of kinematics or dynamic analysis. To generate a moment you need a force multiplied through a distance and the result is a torque. The vehicle resists this torque by its moment of inertia. For the Pitch and Yaw axis this is its moment of inertia. For the Pitch and Yaw axis this is ( $I_L$ ) or lateral moment of inertia. Again we can express this in a form of equations:

For the pitch axis:

$$(1-21) \quad \ddot{\mathbf{q}} = \frac{(Fd)}{I_L} + \mathbf{q}_0 \quad \text{or}$$

$$(1-22) \quad \ddot{\mathbf{q}} = \frac{F_N(\bar{X}_n - X_{cg})}{I_L} + \mathbf{q}_0$$

Where:

$\mathbf{q}$  is the present value of angular acceleration in radians per second squared

$F_N$  is the magnitude of force

$\bar{X}_n$  is the center of pressure location of the vehicle component (nose or fin) in inches

$X_{cg}$  is location of the center of gravity in inches

$\ddot{\mathbf{q}}$  is the initial value of angular acceleration

$I_L$  is the lateral moment of inertia

Since we are now ready to compute the vehicle's angular position, rate, and acceleration, we need to make up a set of coordinates or axis centered on the vehicle's center of gravity.

### Coordinate System

Figure 4 illustrates the coordinate system for the vehicle to be studied. If we add a set of four guidance fins at the nose we will note that these fins oppose the forces generated on the base fins. We will discuss later the use of guidance fins.

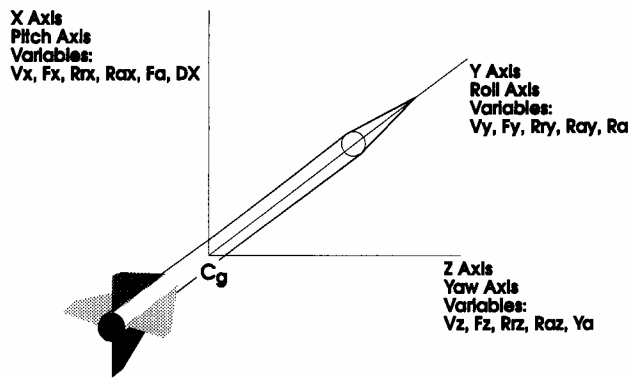


Figure 4

Using the coordinate system shown above, we can now separate the forces, angles, accelerations, and rates of rotation about their related axis.

NOTE 6

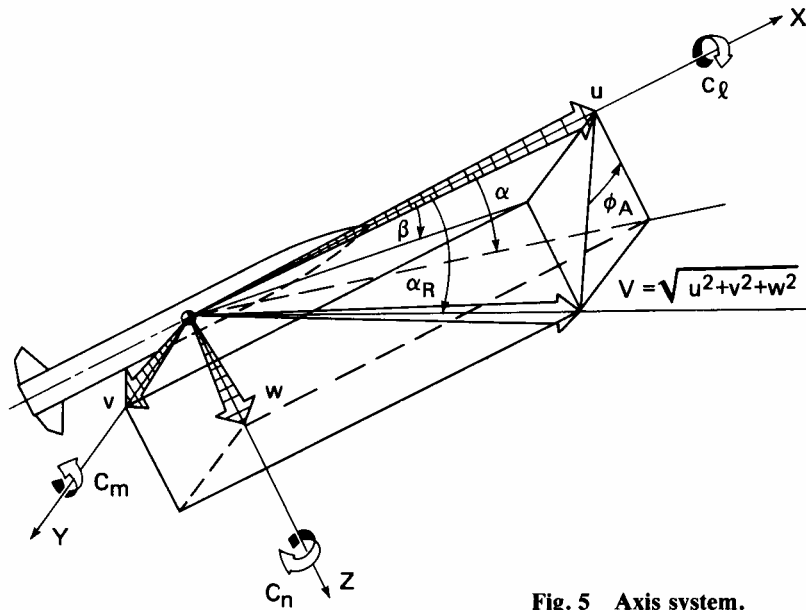


Fig. 5 Axis system.

(Figure taken from “Tactical Missile Aerodynamics: General Topics” edited by Michael J. Hensch. AIAA 1992, page 40)

Here’s another coordinate system that is more of a standard for a rocket in relation to the airstream (body-fixed system). All of the notation in the figure is per NACA standard notation given in Table 1. In this coordinate system X is the longitudinal (roll) axis, Y is the lateral (pitch) axis, and Z is the normal (yaw) axis.  $\alpha_R$  is the angle between the X axis and the total velocity vector which includes angle of attack  $\alpha$  and angle of side slip  $\beta$ .  $\phi_A$  is the aerodynamic roll attitude. The total velocity vector  $V$  is the actual airstream.

For pitch motion we will rotate about the Z axis [Y axis] and for yaw we will use the X axis [Z axis]. The Y axis [X axis] is for roll. This article will not consider the case of a rolling vehicle. Extensive work on rolling vehicles has shown that excess roll reduces altitude performance. In addition, the use of spinerons does not lead to correction of gravity turning but reduces the effect of weather cocking to some degree. In this analysis, then, roll motion about the Y axis [X axis] is not considered, nor is it considered to be an asset in good vehicle in a translational sense. Cross winds will affect the vehicle’s flight path when applied along the pitch and yaw axis (Z and X) [Y

and Z]. This is by a non-zero angle of attack of air flow over the nose, fins, boat tail and transition sections. These produce forces that rotate the rocket by applying torques. In order to determine the vehicles rate of rotation and position at a given instant of time, the set of equations below provide these quantities [for the pitch axis. The equations should be the same for the yaw axis if the rocket is symmetric in its cross-section, i.e. has uniform weight distribution at any given longitudinal point on the rocket, is round, and has at least three fins]:

(1-23)

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_0 + \frac{F_{N(nose)}(X_{cg} - \bar{X}_{cp(nose)})}{I_L} + \frac{2F_{N(fins)}(X_{cg} - \bar{X}_{cp(fins)})}{I_L} + \frac{2F_{N(canards)}(X_{cg} - \bar{X}_{cp(canards)})}{I_L} - C_{m\dot{a}}$$

(1-24)  $\dot{\mathbf{q}} = \dot{\mathbf{q}}_0 + \Delta t \ddot{\mathbf{q}}$

(1-25)  $\mathbf{q} = \mathbf{q}_0 + \Delta t \dot{\mathbf{q}} + \frac{1}{2} \Delta t^2 \ddot{\mathbf{q}}$

Where:

$\ddot{\mathbf{q}}$  is the [pitch] angular acceleration in radians per second squared

$\ddot{\mathbf{q}}_0$  is the initial value of [pitch] angular acceleration

$\dot{\mathbf{q}}$  is the pitch rate in radians per second

$\mathbf{q}$  is the pitch angle in radians

$\dot{\mathbf{q}}_0$  is the initial pitch rate

$\mathbf{q}_0$  is the initial pitch angle in radians

$F_{N(nose)}$  is the [normal] force generated by the nose cone in the pitch axis (Z)

$F_{N(fins)}$  is the base fin [normal] force along the pitch axis

$F_{N(canards)}$  is the guidance fin [normal] force along the pitch axis

$X_{cg}$  is the center of gravity location

$\bar{X}_{cp(nose)}$  is the center of pressure location of the nose

$\bar{X}_{cp(fins)}$  is the lateral center of pressure location for the base fins

$\bar{X}_{cp(canards)}$  is the center of pressure location for the guidance fins

$C_{m\dot{a}}$  is the corrective damping effect on the pitch axis (see discussion below)

For the Yaw axis we use:

(1-26)

$$\ddot{\mathbf{y}} = \ddot{\mathbf{y}}_0 + \frac{F_{N(nose)}(X_{cg} - \bar{X}_{cp(nose)})}{I_l} + \frac{2F_{N(fins)}(X_{cg} - \bar{X}_{cp(fins)})}{I_l} + \frac{2F_{N(canards)}(X_{cg} - \bar{X}_{cp(canards)})}{I_l} - C_{n\dot{\mathbf{b}}}$$

(1-27)  $\dot{\mathbf{y}} = \dot{\mathbf{y}}_0 + \Delta t \ddot{\mathbf{y}}$

(1-28)  $\mathbf{y} = \mathbf{y}_0 + \Delta t \dot{\mathbf{y}} + \frac{1}{2} \Delta t^2 \ddot{\mathbf{y}}$

Where:

$\ddot{\mathbf{y}}$  is the [yaw] angular acceleration in radians per second squared

$\ddot{\mathbf{y}}_0$  is the [yaw] initial value of angular acceleration

$\dot{\mathbf{y}}$  is the yaw rate in radians per second

$\mathbf{y}$  is the yaw angle in radians

$\dot{\mathbf{y}}_0$  is the initial yaw rate

$\mathbf{y}_0$  is the initial yaw angle in radians

$F_{N(nose)}$  is the [normal] force generated by the nose cone in the pitch axis (Z)

$F_{N(fins)}$  is the base fin [normal] force along the pitch axis

$F_{N(canards)}$  is the guidance fin [normal] force along the pitch axis

$X_{cg}$  is the center of gravity location

$\bar{X}_{cp(nose)}$  is the center of pressure location of the nose

$\bar{X}_{cp(fins)}$  is the lateral center of pressure location for the base fins

$\bar{X}_{cp(canards)}$  is the center of pressure location for the guidance fins

$C_{nb}$  is the corrective damping effect on the yaw axis (see discussion below)

Equations (1-23) through (1-28) will thus give all the angular data for vehicle in terms of position, rate and acceleration. Two notable items are in the above equations. The first of these is that the moment arm produced by the guidance fins ( $\bar{X}_{cp(canards)} - X_{cg}$ ) is a negative term and will normally be a negative number. This is due to the fact the guidance fins are located ahead of the center of gravity of vehicle. This in turn makes the guidance fin effects act to counteract the effects of the fins at the base of the vehicle. The second notable aspect of the above equations are the terms  $C_{m\dot{a}}$  and  $C_{m\dot{b}}$ . These terms are due to the damping effect of the rocket as it rotates and is discussed next.

### **Oscillation and Rotation Dampening [Rotation Damping]**

As a vehicle rotates through its pitch and yaw axis due to a change of air flow over the fins (changes in angle of attack), or due to a gravity turning effect, a dampening effect that will reduce the rotation acceleration on the vehicle and, in turn, the rotation rate. In mechanical and electronic engineering, dampening is a very important aspect of system behavior. For us, in the performance of dynamic flight analysis, rotational dampening must be considered.

As the vehicle rotates about its center of gravity, a dampening effect takes place due to the rate of rotation. The vehicle rotation rate acts to reduce the angle of attack of the vehicle and thus the duration and amplitude of oscillations. In terms of an equation we find the dampening coefficient ( $D_c$ ) to be:

(1-29)

$$D_c = \frac{1}{2} \mathbf{r} V^2 \mathbf{a} \left( C_{Na(nose)} (\bar{X}_{cp(nose)} - X_{cg})^2 + C_{Na(fins)} (\bar{X}_{cp(fins)} - X_{cg})^2 + C_{Na(canards)} (\bar{X}_{cp(canards)} - X_{cg})^2 \right)$$

Where:

$D_c$  is the dampening coefficient

$A_r$  is the cross sectional area of the rocket in inches(squared)(reference area)

$V$  is the velocity of the rocket in FPS (see equation (1-23) for the remaining variable names)

From the above equation, we observe that the rocket will have a greater dampening (as the velocity increases linearly, so will the dampening coefficient). The locations of the component parts' center of pressure and the location of the center of gravity strongly effect the magnitude of the dampening coefficient as a square of the distance between them. In terms of vehicle design when good dampening is desired, one will find a very short length behind the center of gravity (about one vehicle diameter aft of the  $X_{cg}$ ) where vehicle dampening fits a nearly perfect quarter wave dampening effect.

Looking back at equations (1-23) and (1-26) we mentioned the variables  $C_{m\dot{a}}$  and  $C_{m\dot{b}}$  known as the corrective dampening effect. If we take the dampening coefficient ( $D_c$ ) and multiply it by the rate of rotation on an axis and then divide by the moment of inertia ( $I_L$ ) we get the value for  $C_{m\dot{a}}$  and  $C_{n\dot{b}}$ . Equations (1-30) and (1-31) illustrate this:

$$(1-30) \quad C_{m\dot{a}} = \frac{D_c \dot{\mathbf{a}}}{I_L}$$

$$(1-31) \quad C_{n\dot{b}} = \frac{D_c \dot{\mathbf{b}}}{I_L}$$

We now have a complete set of formulas to tackle the problem of a dynamic simulation of a rocket, be it guided or unguided. In the form of a computer program, we are now able to see the effects of gravity turning which causes a ballistic trajectory and the effect of weather cocking and how a rocket responds to these effects. What is left to accomplish is to relate the velocity along all three axis to the angle of flight and [\[angle](#)

of] attack of the fins. Since we are primarily concerned with the angular rotations and flight path of a vehicle, we will use an approximation of average thrust and burn time for our simulation. Other programs offer very precise solutions to trajectory analysis such as those from Rogers Aerospace or the AeroSoft Ltd. Program MROC8. By very precise I mean that vehicle speed and altitude data having errors less than 2% of the actual flight. In order to determine the velocity along the three axis of a vehicle we can use the following equations:

$$(1-32) \quad V_y = V_{y0} + \frac{(F - W)\Delta t}{\sqrt{M_b^2 + K\Delta t^2(F - W)}} - g\Delta t$$

$$(1-33) \quad V_x = V_{x0} + V_y \text{Sin} \mathbf{q}$$

$$(1-34) \quad V_z = V_{z0} + V_y \text{Sin} \mathbf{y}$$

$$(1-35) \quad \mathbf{a} = \text{Tan}^{-1} \frac{(V_z + V_{wz})}{V_y} - \mathbf{q}$$

$$(1-35) \quad \mathbf{b} = \text{Tan}^{-1} \frac{(V_x + V_{wx})}{V_y} - \mathbf{y}$$

Where:

$V_{y0}$  is the initial velocity in the Y axis

$V_{x0}$  is the initial velocity in the X axis

$F$  is the average thrust of the rocket motor

$W$  is the initial weight in pounds of the rocket

$\Delta t$  is the time interval in seconds of the total burn time and is usually .005 to .001 seconds

$M_b$  is the burnout weight of the vehicle divided by  $g$  (32.174 ft/s<sup>2</sup>)

$K$  is the drag coefficient found by:  $K = \frac{1}{2} \rho A_r$

[  $A_r$  is the reference cross-sectional area, i.e. rocket cross-sectional area]

$q$  is the Pitch Angle

$y$  is the yaw Angle

$a$  is the angle of attack along the Pitch Axis

$b$  is the angle of attack along the Yaw axis (angle of sideslip)

$V_{wz}$  is the cross wind velocity in FPS along the Z axis

$V_{wx}$  is the cross winds velocity in FPS along the X axis

#### NOTE 7

This is a little confusing.  $V_y$  is defined as if it were in a body-fixed coordinate system, but  $V_x$  and  $V_z$  are defined as if they were space-fixed. If you were to measure the velocities along all three axes using onboard accelerometers, the velocities would all be determined referred to a body-fixed coordinate system as in the figure in Note 6. Then the angles could all be easily determined as:

$$a = \text{Tan}^{-1} \frac{w}{u}$$

$$b = \text{Tan}^{-1} \frac{v}{u}$$

$$f_A = \text{Tan}^{-1} \frac{v}{w}$$

To convert the body fixed system to a space-fixed one in all three axes, a matrix transformation using the Euler angles must be performed, which is best done with a program like Matlab.

What is important to realize in using the above functions is that we are forced to shuffle the order of equations around within a program. The reason for this is based on

the fact that in order to find the forces generated by the fins due to the normal force coefficient we need a value of  $\mathbf{a}$  and  $\mathbf{b}$  (angles of attack). This must be performed first in order to then determine the rocket's rate of rotation and acceleration rate of rotation (both of which effect the angles of pitch and yaw). From a programming standpoint this forces us to keep the time interval ( $\mathbf{Dt}$ ) very small in value in the range of .001 to .005 seconds. The textbook Advanced Topics in Model Rocketry (MIT Press) stress this fact, and software testing of the flight dynamics simulator showed the need for keeping the time interval small.

This concludes the Flight Mechanics portion of this article. the complete set of 36 equations can account for the effects of gravity turning and weather cocking in the pitch and yaw axis in a non-rolling vehicle. Before discussing the software simulator that was produced using these equations, we need to talk about active guidance systems that can be flown high performance model rockets that can lift approximately five to eight ounces of payload.

#### NOTE 8

This article goes on to discuss ways in which a guidance system could be built for an amateur rocket using a pendulum as an orientation sensor. Unfortunately this is wrong; a pendulum cannot solely be used, so I am leaving that part of the article out. The pendulum argument has come up so many times it even has a name now: the pendulum fallacy. The author of this article is not the first to run up on the rocks when thinking about pendulums. Robert Goddard, in the early days, ran afoul of this as well. We have all seen the famous picture of Robert Goddard standing next to one of his first rockets, posed next to an open frame rocket in the snow. If you look carefully you'll see that Goddard's rocket has the motor on top of the propellant tanks! Another version of the pendulum fallacy.

#### References

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2. [http://www.apogeerockets.com/education/downloads/Barrowman\\_report.pdf](http://www.apogeerockets.com/education/downloads/Barrowman_report.pdf)
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